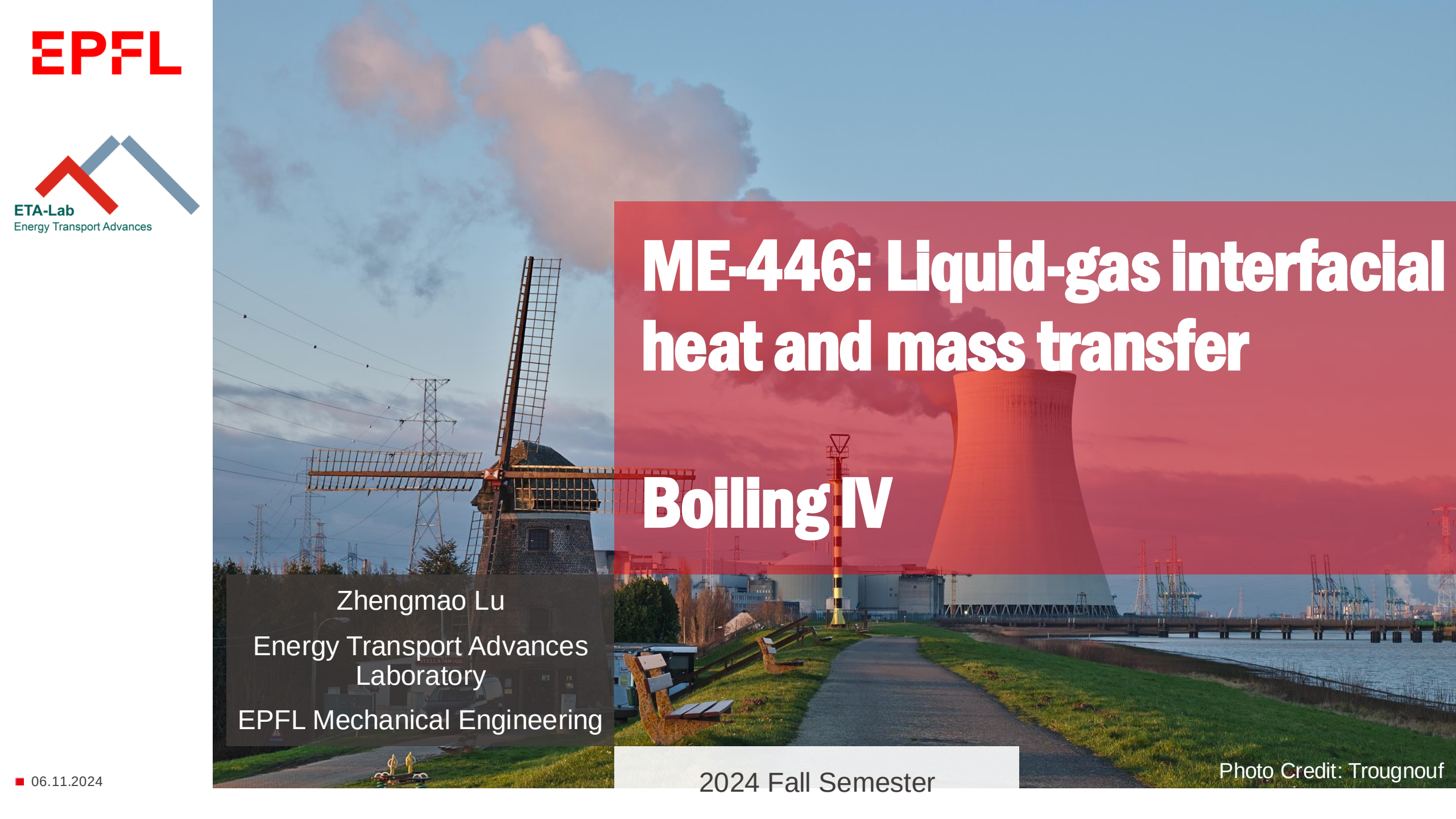


The background image is a composite of two scenes. The left side shows a traditional wooden windmill with a lattice-structured sail, situated in a grassy field with a paved path leading towards it. In the background, there are several high-voltage electrical transmission towers. The right side of the image shows a large industrial facility, likely a power plant, with a prominent white cooling tower emitting a plume of white steam. The sky is a mix of blue and light orange, suggesting a sunset or sunrise. A semi-transparent red rectangular box is overlaid on the right side of the image, containing the main title and subtitle in white text.

ME-446: Liquid-gas interfacial heat and mass transfer

Boiling IV

Zhengmao Lu
Energy Transport Advances
Laboratory
EPFL Mechanical Engineering

2024 Fall Semester

Photo Credit: Trougnouf

- Bubble departure frequency and diameter
- Different regimes in pool boiling
- Rohsenow's microconvection model for nucleate boiling

- Zuber's CHF model based on Helmholtz and Taylor instabilities
- Force balance model for CHF
- Effect of surface parameters on CHF
- Statistical approach for CHF

Helmholtz Instability of Vapor Columns

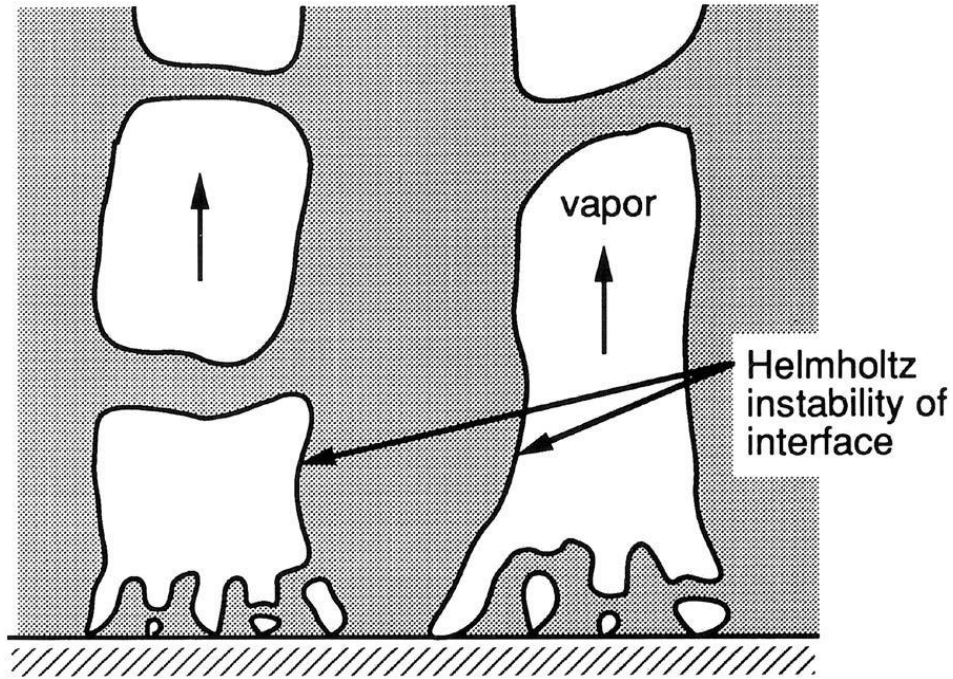
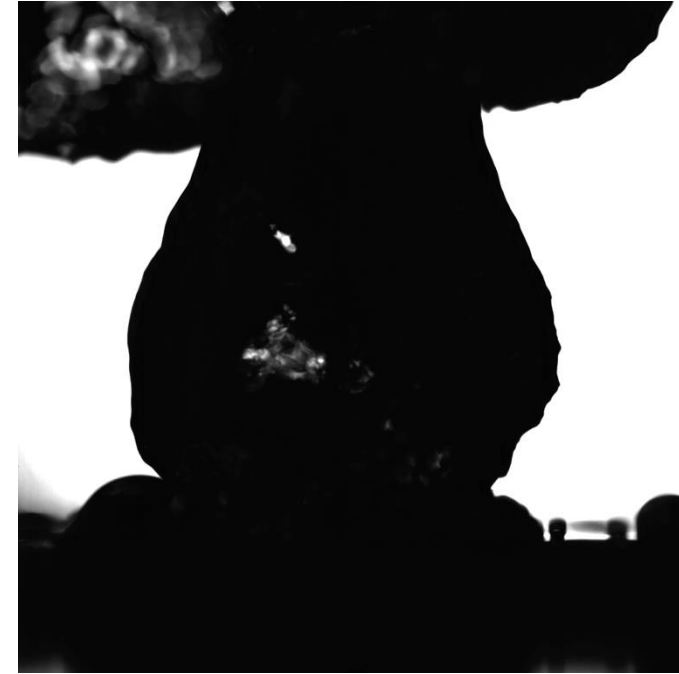


Figure 7.16 Carey



Video credit: Dr. Rameez Iqbal

Vapor columns form at high heat fluxes

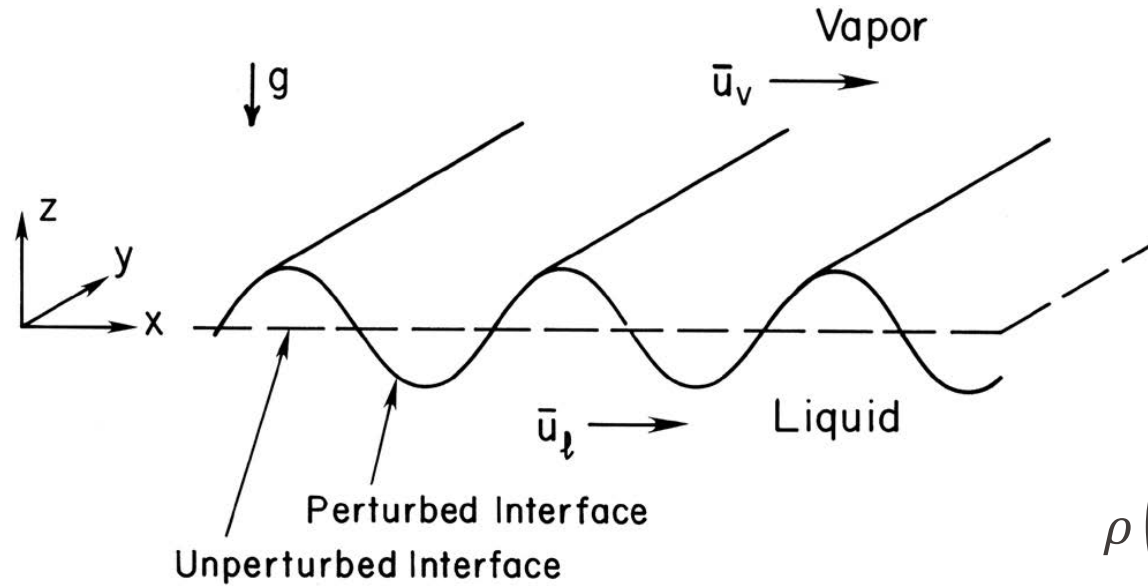


Figure 4.4 in Carey

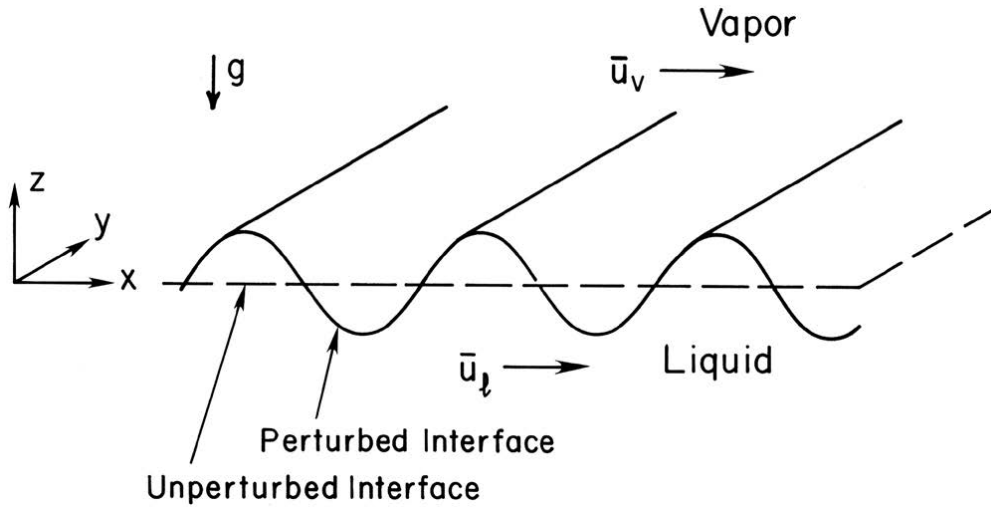
Continuity

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

Momentum balance
(neglecting viscosity)

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} \right) = - \frac{\partial P}{\partial x}$$

$$\rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} \right) = - \frac{\partial P}{\partial z} - \rho g$$



Consider after a perturbation $\delta(x, t = 0) = Ae^{i\alpha x}$

$$u: \bar{u} \rightarrow \bar{u} + u', \quad w: 0 \rightarrow w', \quad P: \bar{P} \rightarrow \bar{P} + P'$$

$$\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0$$

$$\rho \left(\frac{\partial u'}{\partial t} + \bar{u} \frac{\partial u'}{\partial x} \right) = - \frac{\partial P'}{\partial x} \quad \Rightarrow \frac{\partial^2 P'}{\partial x^2} + \frac{\partial^2 P'}{\partial z^2} = 0$$

$$\rho \left(\frac{\partial w'}{\partial t} + \bar{u} \frac{\partial w'}{\partial x} \right) = - \frac{\partial P'}{\partial z}$$

Postulate the form of the response function:

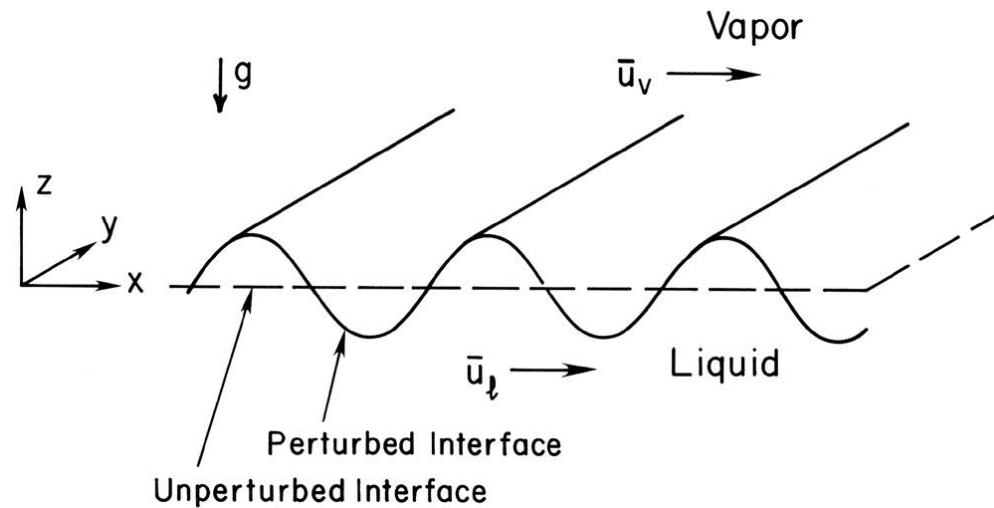
$$\delta = Ae^{i\alpha x + \beta t}$$

$$w' = \hat{w}(z)e^{i\alpha x + \beta t}$$

$$P' = \hat{P}(z)e^{i\alpha x + \beta t}$$

We are interested in β

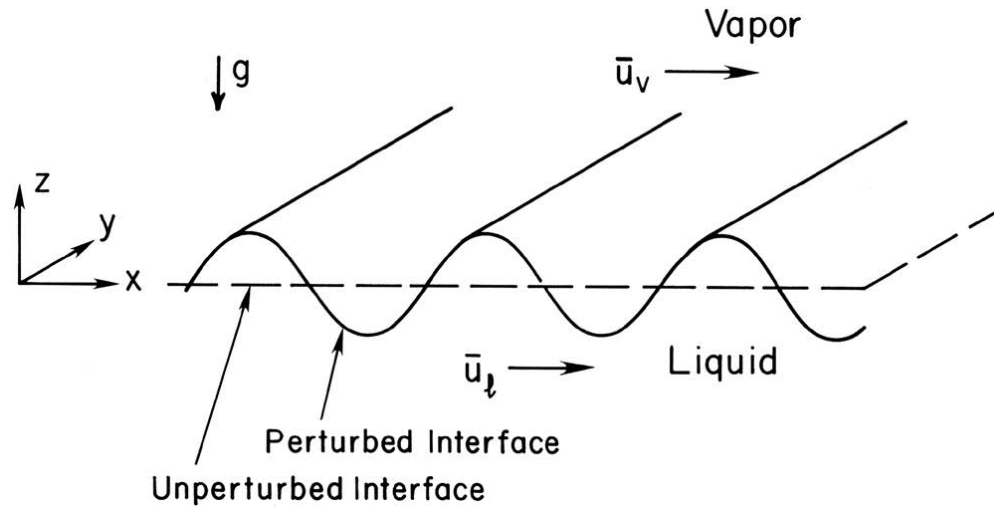
$$\frac{\partial^2 P'}{\partial x^2} + \frac{\partial^2 P'}{\partial z^2} = 0 \quad P' = \hat{P}(z)e^{i\alpha x + \beta t}$$



$$w' = \hat{w}(z)e^{i\alpha x + \beta t} \quad \rho \left(\frac{\partial w'}{\partial t} + \bar{u} \frac{\partial w'}{\partial x} \right) = - \frac{\partial P'}{\partial z}$$

Helmholtz Instability

$$\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0 \quad w' = \hat{w}(z)e^{i\alpha x + \beta t}$$



$$\delta = Ae^{i\alpha x + \beta t}$$

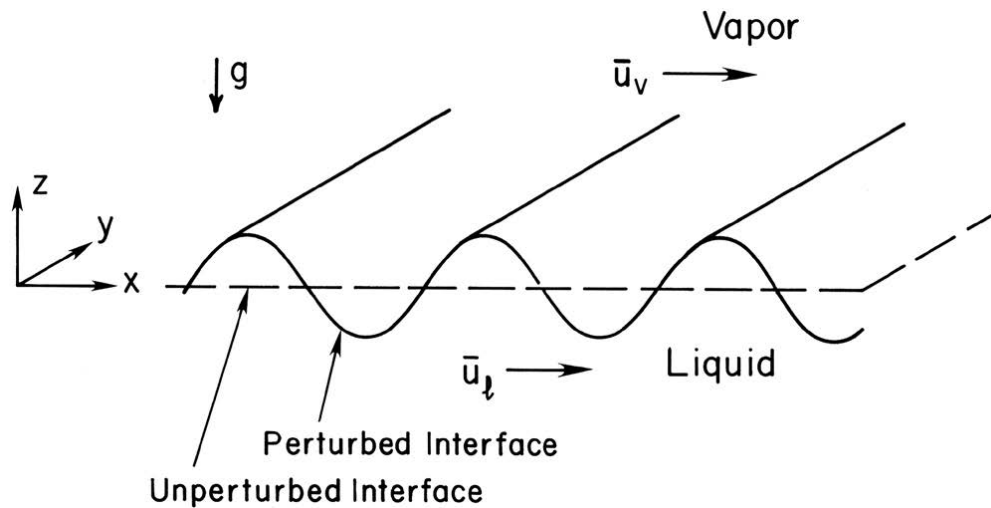
$$\hat{w}_v(z) = \frac{a_v \alpha}{\rho_v (\beta + i\alpha \bar{u})} e^{-\alpha z}$$

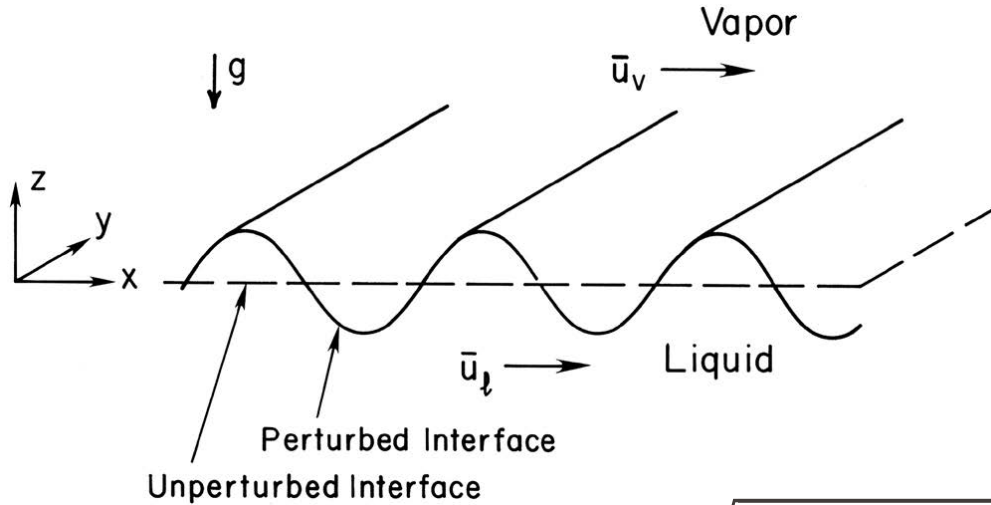
$$\hat{w}_l(z) = -\frac{a_l \alpha}{\rho_l (\beta + i\alpha \bar{u})} e^{\alpha z}$$

$$a_v = \frac{\rho_v}{\alpha} (\beta A + i\alpha \bar{u}_v)^2 A$$

$$a_l = -\frac{\rho_l}{\alpha} (\beta A + i\alpha \bar{u}_l)^2 A$$

$$\hat{P}_v = a_v e^{-\alpha z} \quad \hat{P}_l = a_l e^{\alpha z}$$





$$\text{Perturbation } \delta(x, t = 0) = Ae^{i\alpha x}$$

$$\delta = Ae^{i\alpha x + \beta t}$$

$$w' = \hat{w}(z)e^{i\alpha x + \beta t}$$

$$p' = \hat{p}(z)e^{i\alpha x + \beta t}$$

$$\beta = \pm \frac{\sqrt{\alpha^2 \rho_v \rho_l (\bar{u}_v - \bar{u}_l)^2 - (\sigma \alpha^3 + \Delta \rho g \alpha)}}{\rho_v + \rho_l} - i\alpha \frac{\rho_l \bar{u}_l + \rho_v \bar{u}_v}{\rho_v + \rho_l}$$

The perturbation will cause a growing response if and only if β has a positive real part

Instability condition:

$$|\bar{u}_v - \bar{u}_l| > \sqrt{\frac{\left(\sigma \alpha + \frac{\Delta \rho g}{\alpha}\right) (\rho_l + \rho_v)}{\rho_l \rho_v}}$$

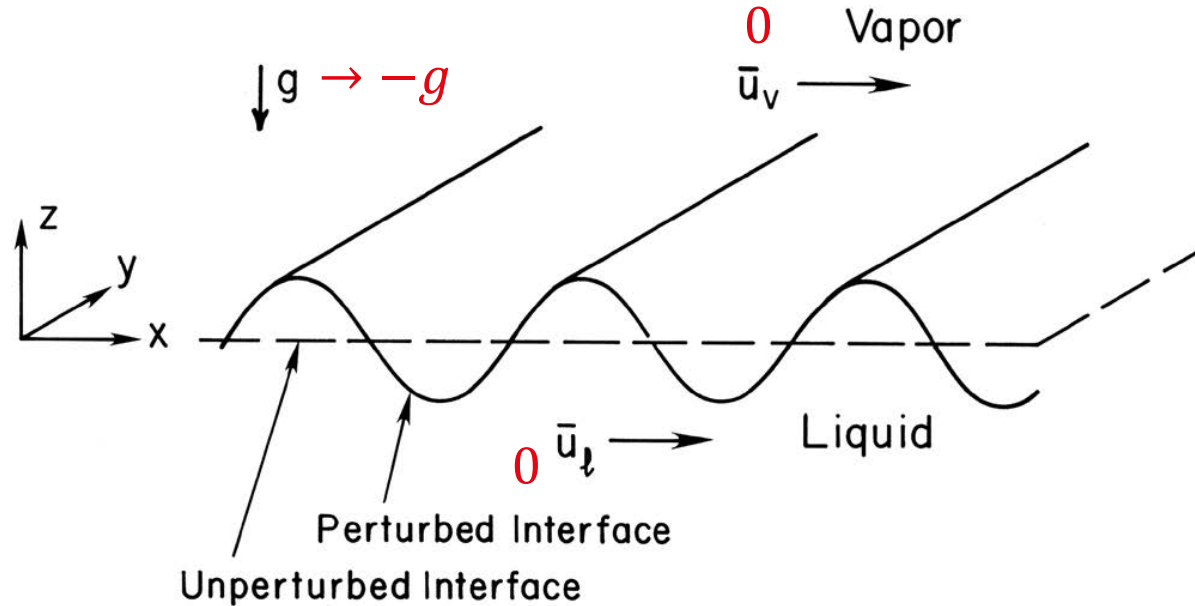
$\bar{u}_v - \bar{u}_l$ promotes instability while gravity and surface tension suppressing instability, we can adjust the value of g based on the orientation of the system.



Helmholtz Instability



Facebook/ Rachel Gordon

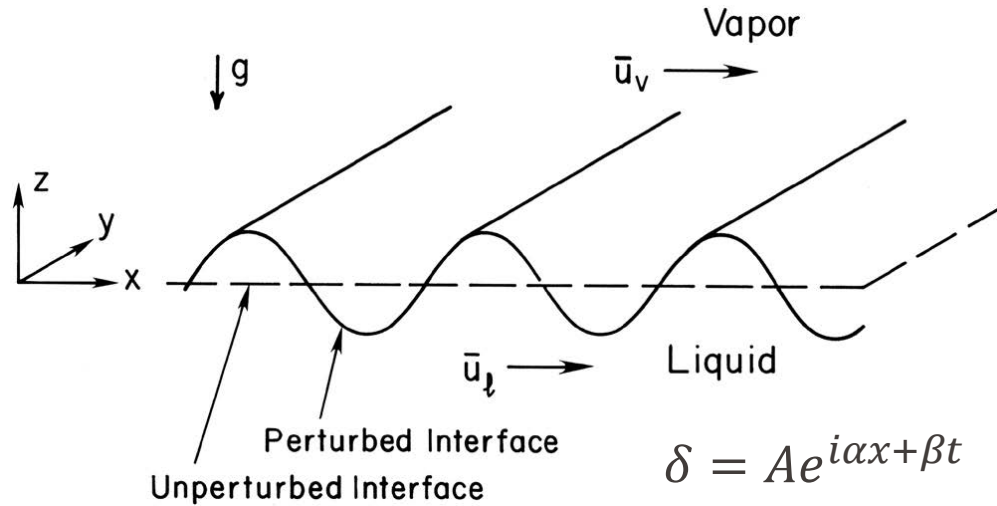


$$\delta = Ae^{i\alpha x + \beta t}$$

$$\beta = \pm \frac{\sqrt{\alpha^2 \rho_v \rho_l (\bar{u}_v - \bar{u}_l)^2 - (\sigma \alpha^3 + \Delta \rho g \alpha)}}{\rho_v + \rho_l} - i\alpha \frac{\rho_l \bar{u}_l + \rho_v \bar{u}_v}{\rho_v + \rho_l} \Rightarrow \beta = \pm \sqrt{\frac{\Delta \rho g \alpha - \sigma \alpha^3}{\rho_l + \rho_v}}$$

The fastest growing perturbation (α_{max}) in this case can be found by setting $\frac{d\beta}{d\alpha} = 0$

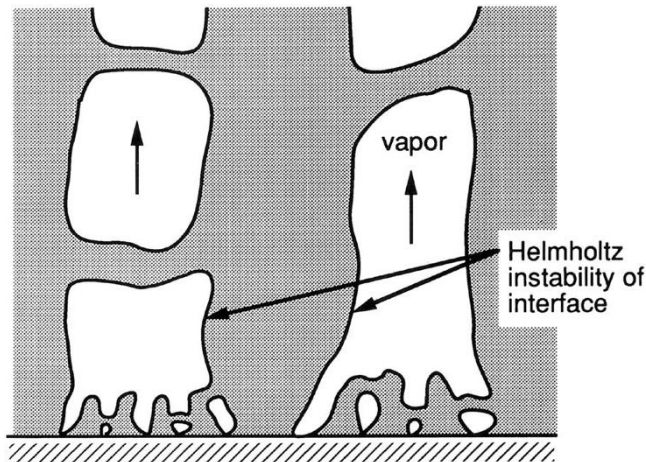
The corresponding most dangerous wavelength $\lambda_D = \frac{2\pi}{\alpha_{max}} = 2\pi \sqrt{\frac{3\sigma}{\Delta \rho g}}$



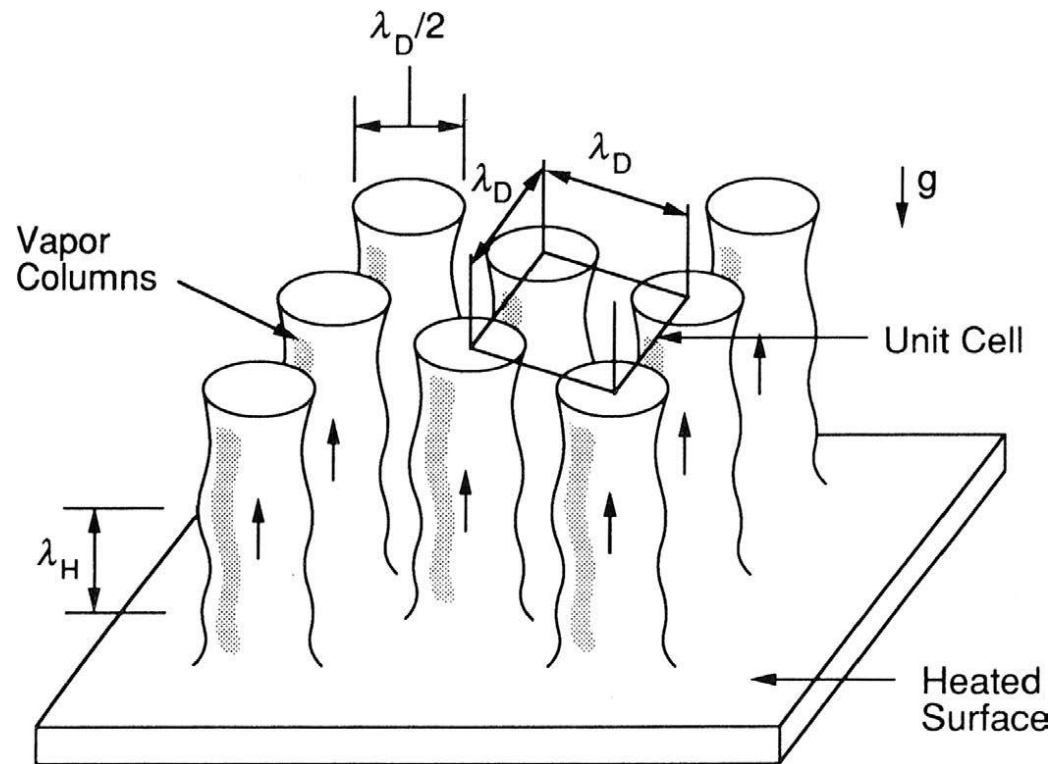
Helmholtz Instability

$$|\bar{u}_v - \bar{u}_l| > \sqrt{\frac{\left(\sigma \alpha + \frac{\Delta \rho g}{\alpha}\right) (\rho_l + \rho_v)}{\rho_l \rho_v}}$$

Setting $g = 0$ for vertical interfaces



$$|\bar{u}_v - \bar{u}_l| > \sqrt{\frac{\sigma \alpha (\rho_l + \rho_v)}{\rho_l \rho_v}} = \sqrt{\frac{2 \pi \sigma (\rho_l + \rho_v)}{\rho_l \rho_v \lambda_H}} = u_c$$



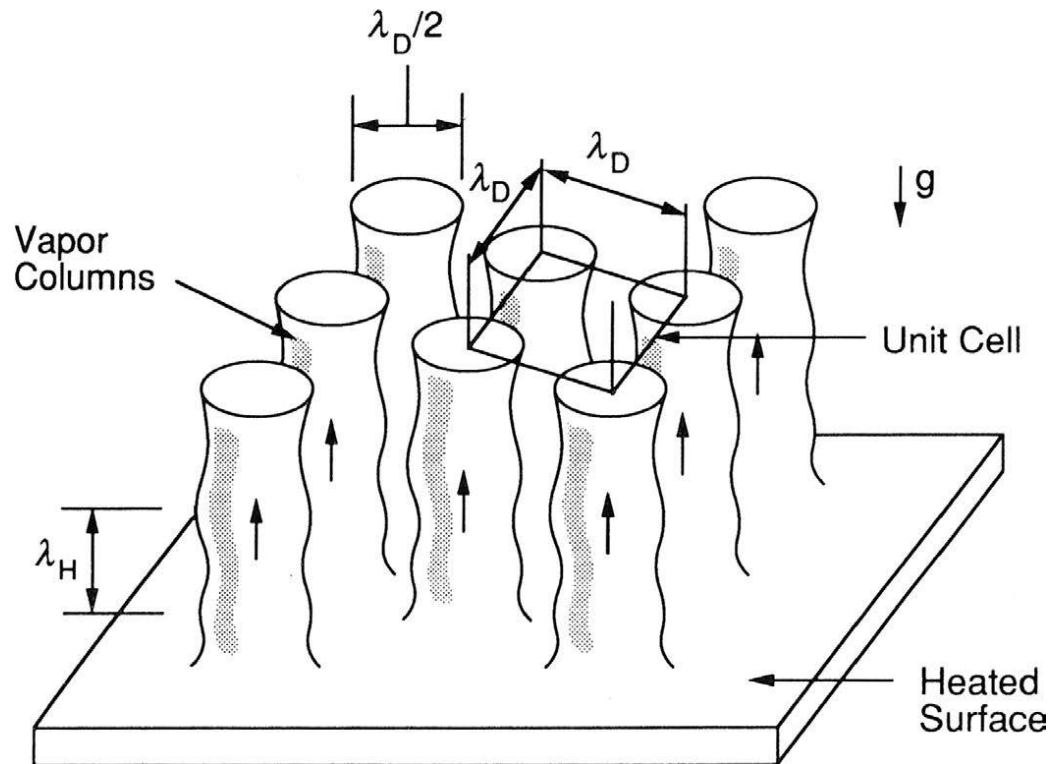
- CHF is reached when interface of vapor columns becomes Helmholtz unstable (λ_H)
- The pitch of the vapor columns coincides with the most dangerous wavelength in Taylor instability

$$\lambda_D = 2\pi\sqrt{3\sigma/\Delta\rho g}$$

- The diameter of vapor column is $\lambda_D/2$

$$\lambda_H = \lambda_D \Rightarrow u_c = \sqrt{\frac{2\pi\sigma(\rho_l + \rho_v)}{\rho_l\rho_v\lambda_H}} \approx \sqrt{\frac{2\pi\sigma}{\rho_v\lambda_D}}$$

$$u_c = \sqrt{\frac{2\pi\sigma}{\rho_v\lambda_D}} \quad \lambda_D = 2\pi \sqrt{\frac{3\sigma}{\Delta\rho g}}$$



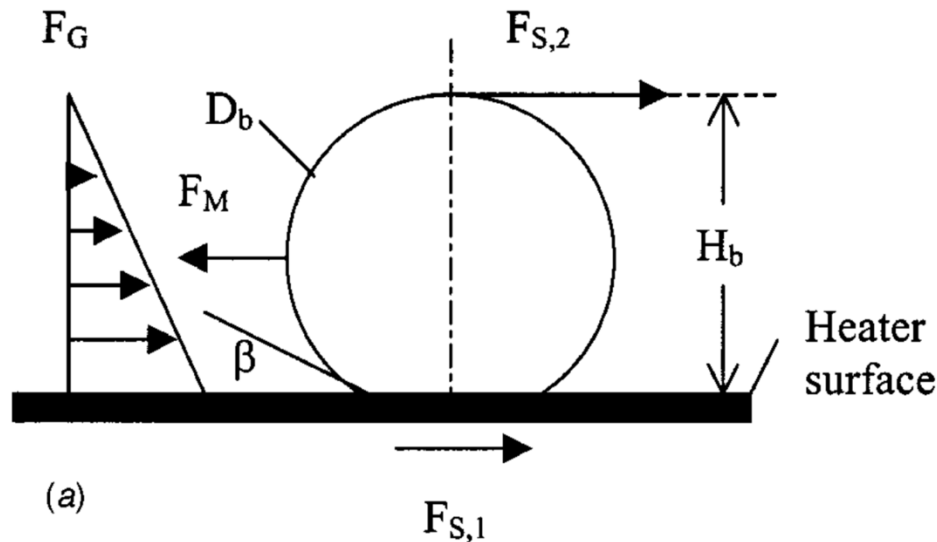
$$u_c = \frac{q''_{max}}{\rho_v h_{lv}} \left(\frac{A_{surf}}{A_{col}} \right) = \frac{16}{\pi} \frac{q''_{max}}{\rho_v h_{lv}}$$

$$q''_{max} = 0.149 \rho_v h_{lv} \left(\frac{\sigma \Delta \rho g}{\rho_v^2} \right)^{1/4}$$

- No way to accommodate effects from geometry and surface wettability
- No clear justification for the choice of vapor column diameter as $\lambda_D/2$
- No visual observation of Helmholtz instability during boiling to date
- Still widely used as a reference model for all subsequence CHF models

Lateral Force Balance Model

<https://doi.org/10.1115/1.1409265>



Considering the lateral direction

Surface tension force : $F_{S,1}$, $F_{S,2}$

Hydrostatic force: F_G

Momentum force: F_M

Kandlikar suggested momentum balance requires $F_{S,1} + F_{S,2} + F_G = F_M$

chose a bubble diameter at CHF $D_b = \lambda_D/2$

λ_D : the most dangerous wavelength in Taylor instability

and set a bubble influence area πD_b^2

$$q_K'' = \rho_v h_{fg} \left(\frac{1 + \cos \beta}{16} \right) \left[\frac{2}{\pi} + \frac{\pi}{4} (1 + \cos \beta) \right]^{\frac{1}{2}} \left(\frac{\sigma \Delta \rho g}{\rho_v^2} \right)^{1/4} \quad (\text{Horizontal surface})$$

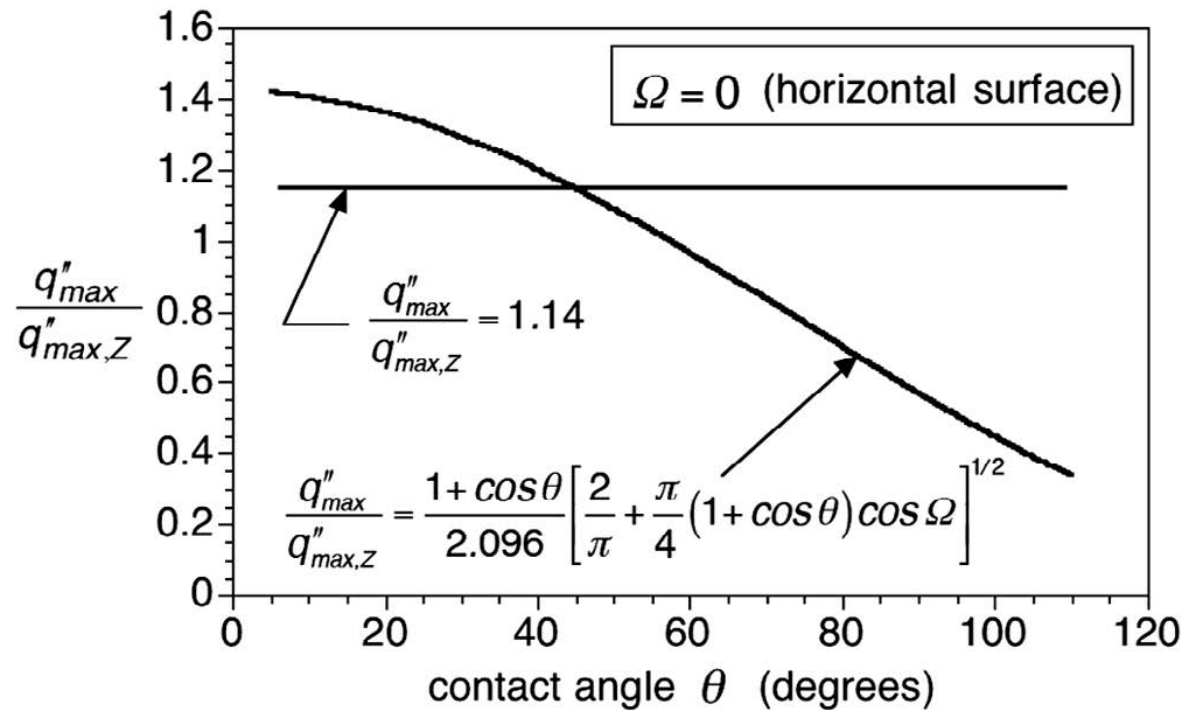
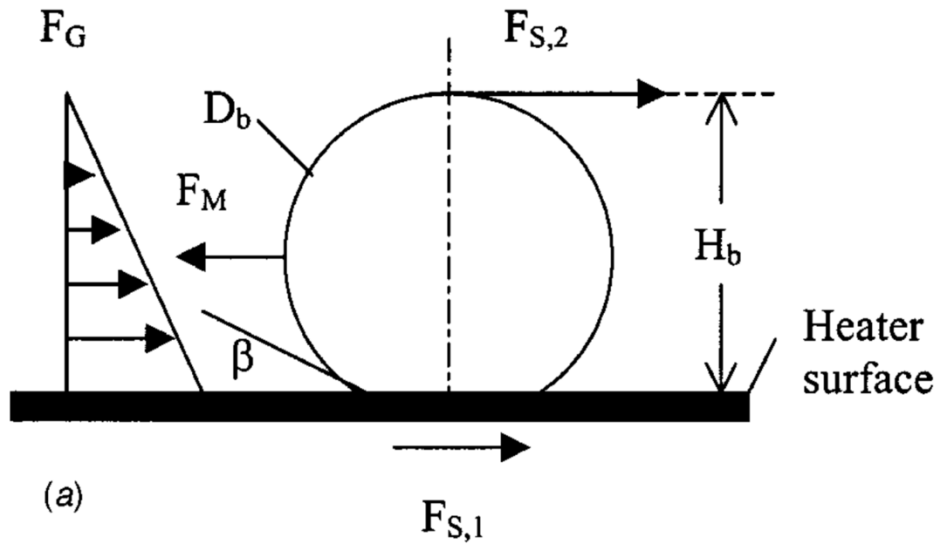


FIGURE 7.19 in Carey

$$q''_{max,Z} = 0.149 \rho_v h_{lv} \left(\frac{\sigma \Delta \rho g}{\rho_v^2} \right)^{1/4}$$

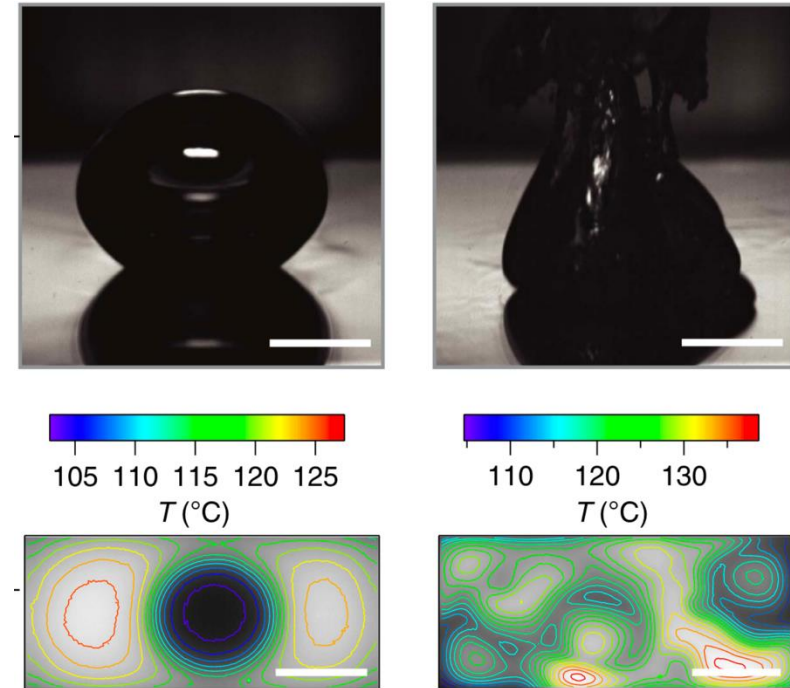
$$\frac{q_K''}{q''_{max,Z}} = \frac{1 + \cos \theta}{2.096} \left[\frac{2}{\pi} + \frac{\pi}{4} (1 + \cos \beta) \right]^{\frac{1}{2}}$$



$$F_{S,1} + F_{S,2} + F_G = F_M$$

Liquid-vapor pressure difference not accounted for in momentum balance

Dhillon *et al.*, *Nat Commun* 2015

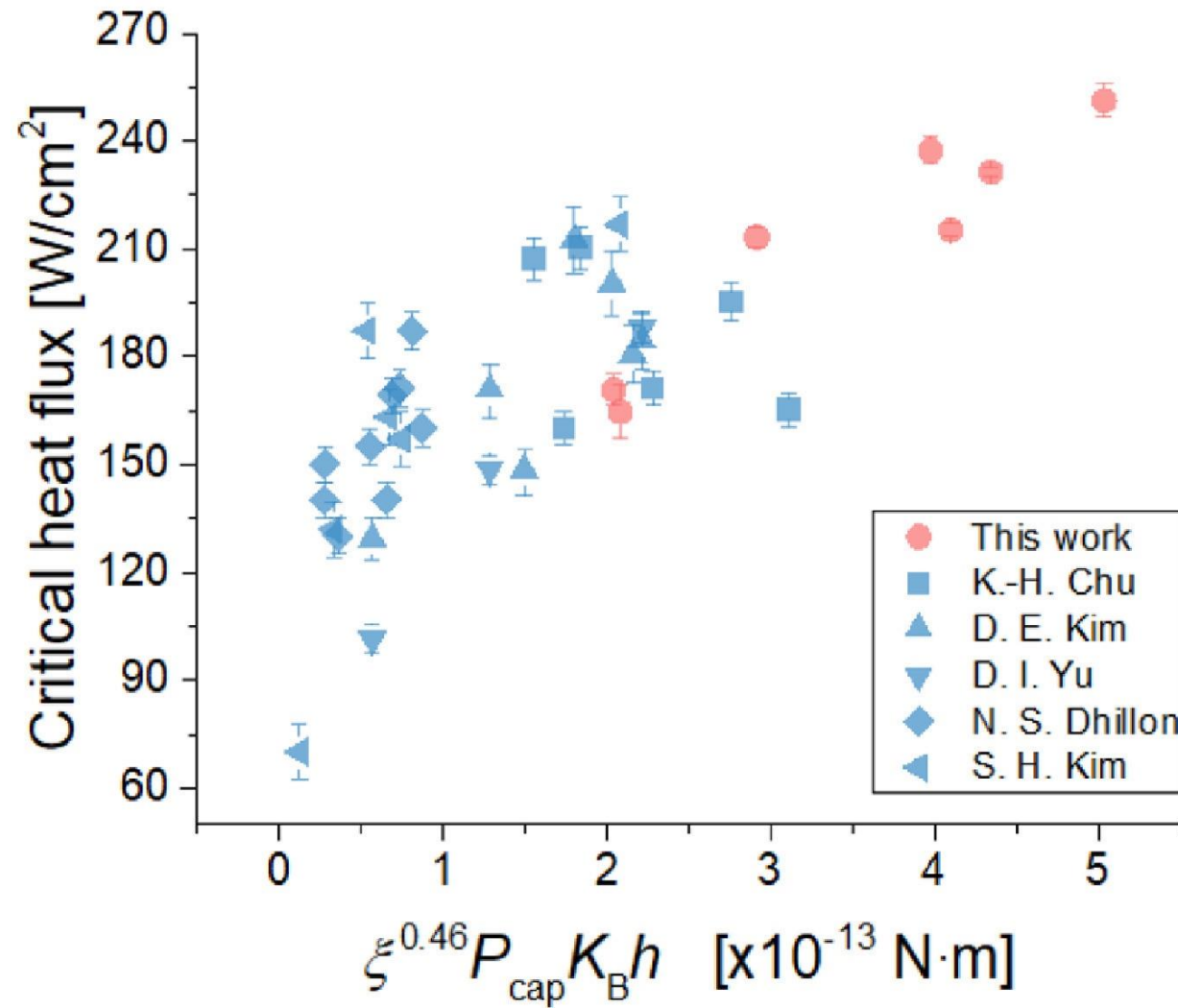


Not clear how geometric parameters are chosen at CHF



<https://doi.org/10.1021/acs.langmuir.7b01522>

CHF vs Wicking on Hemispreading Surfaces

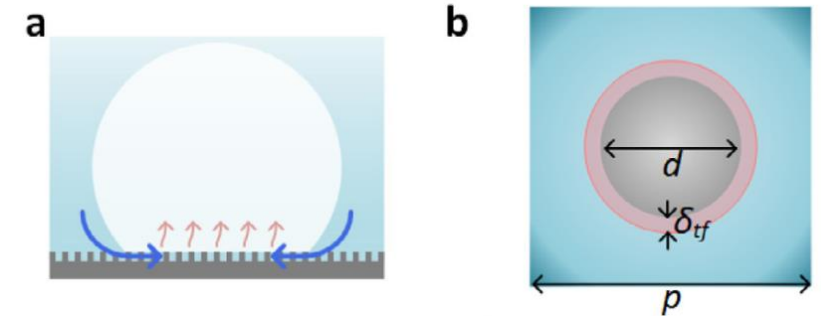


P_{cap} : capillary pressure

K_B : permeability

h : thickness of the wick

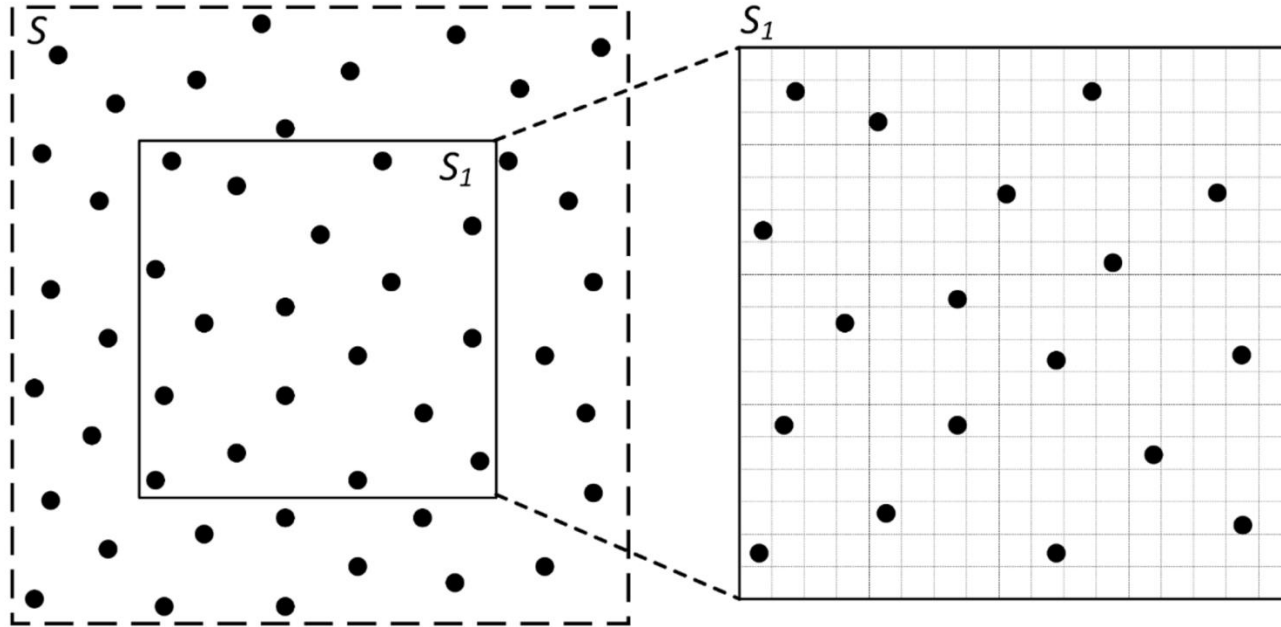
ξ : "thin film" liquid fraction



<https://doi.org/10.1016/j.ijheatmasstransfer.2021.122189>

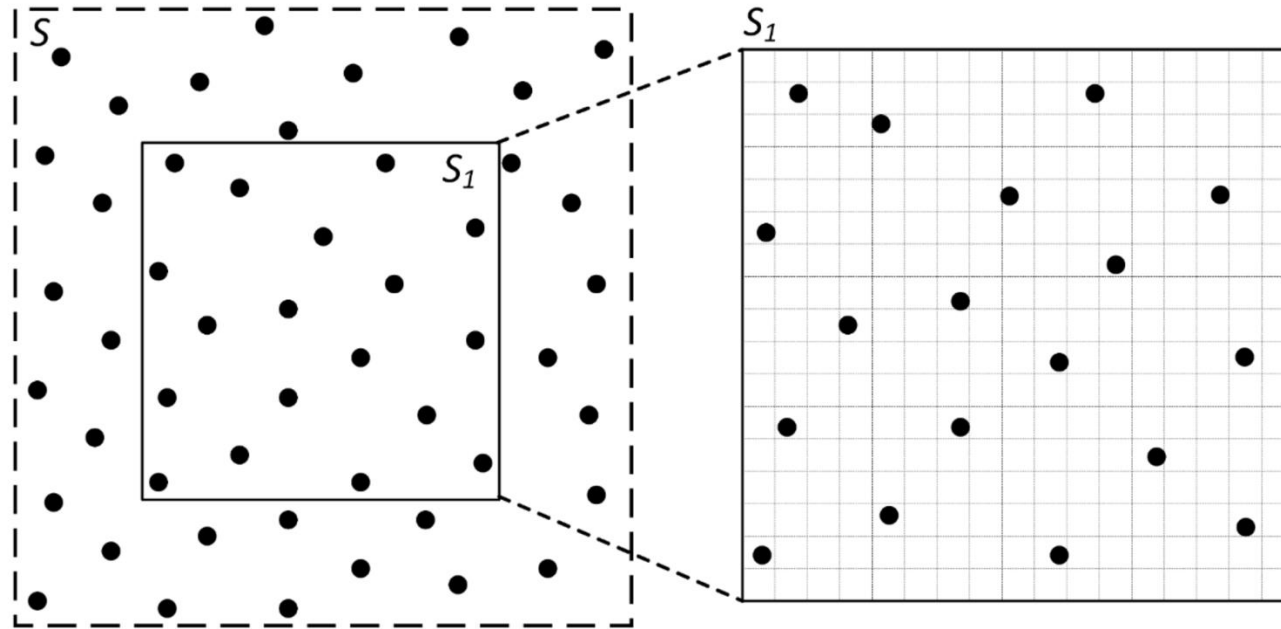
Statistical Approach for Flat Surface Boiling



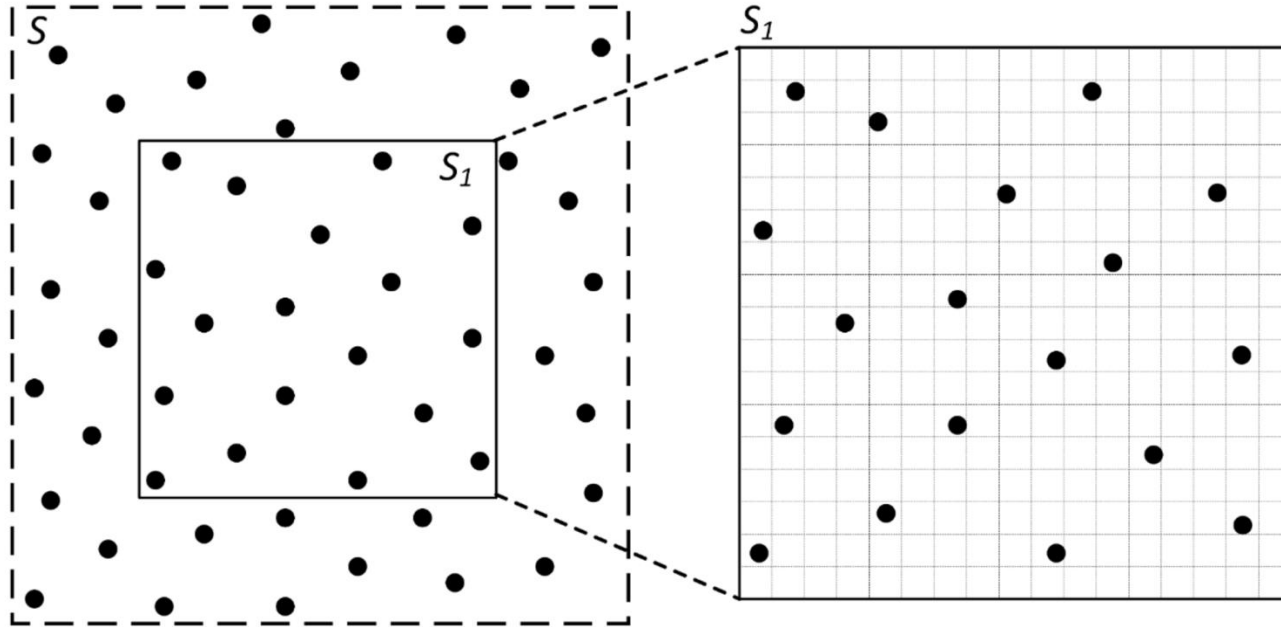


<https://doi.org/10.1016/j.ijheatmasstransfer.2021.121904>

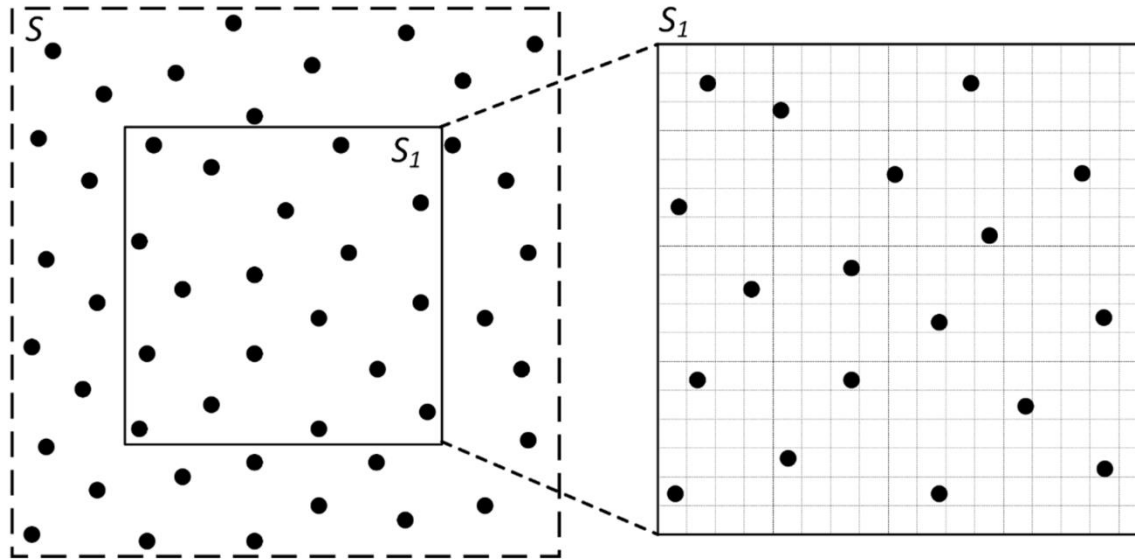
- Consider a large surface S (large enough to ignore edge effects)
- Probability of each point on the surface becoming an active nucleation sites is equal



- Average nucleation density n_0 [m^{-2}]
- For an arbitrary segment of the surface S_1 of area A , the average number of nucleation sites is $N_0 = n_0 A$
- The actual number of nucleation sites in S_1 , $\mathbf{N}$, is a random variable with an expectation value N_0

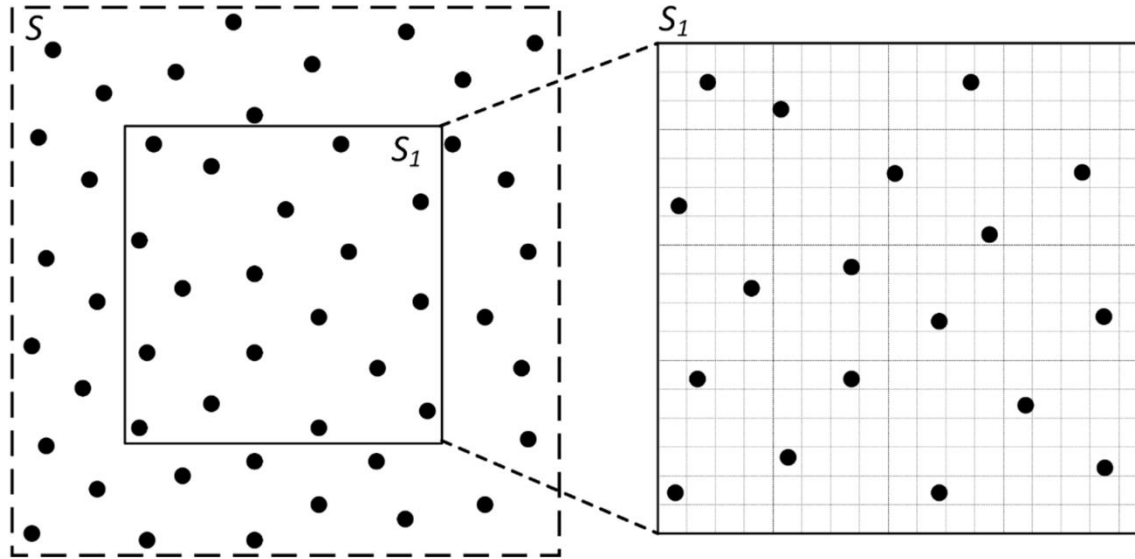


- Divide S_1 into M subsegments with the same area A/M
- Make M large enough such that the number of nucleation sites in each square is 0 or 1.
- Probability of finding a nucleation site in one square $p=N_0/M$



- Probability of find N squares that contain nucleation sites is a binomial distribution

$$\begin{aligned}
 P(N, N_0, M) &= \frac{M!}{N! (M - N)!} p^N (1 - p)^{M - N} \\
 &= \frac{1}{N!} \cdot \frac{M!}{(M - N)! M^N} \cdot N_0^N \left(1 - \frac{N_0}{M}\right)^{M - N}
 \end{aligned}$$



$$P(N, N_0, M) = \frac{1}{N!} \cdot \frac{M!}{(M - N)! M^N} \cdot N_0^N \left(1 - \frac{N_0}{M}\right)^{M-N}$$

Stirling's approximation: $\ln(n!) = n \ln n - n + O(\ln n)$ for $n \rightarrow \infty$

$$\lim_{M \rightarrow \infty} \ln \left[\frac{M!}{(M - N)! M^N} \right] = 0 \Rightarrow \lim_{M \rightarrow \infty} \frac{M!}{(M - N)! M^N} = 1$$

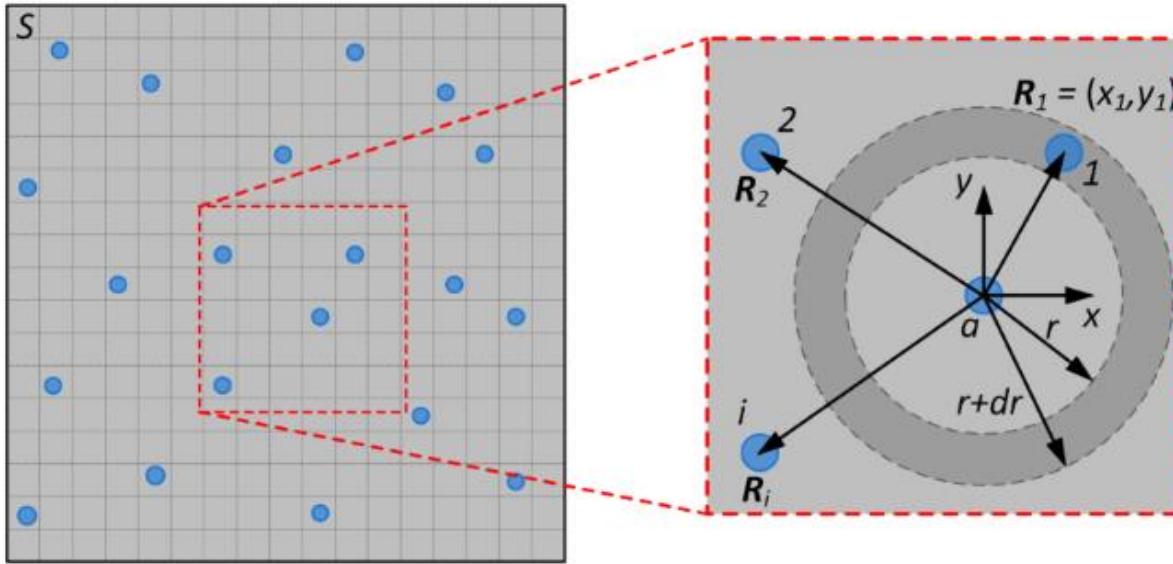
$$\lim_{M \rightarrow \infty} P(N, N_0, M) = \frac{N_0^N}{N!} e^{-N_0}$$

$$\text{Po}(N, N_0) = \frac{N_0^N}{N!} e^{-N_0}$$

$$\sum_{N=0}^{\infty} P(N, N_0) = \sum_{N=0}^{\infty} \frac{N_0^N}{N!} e^{-N_0} = e^{N_0} \cdot e^{-N_0} = 1$$

- Isolated bubbles dissipate heat better than merged bubbles
- CHF is reached when you have the maximum number of isolated bubbles
- With elevated temperature, more nucleation sites become activated while more bubbles are likely to merge into each other.
- It is important to consider the distance between bubbles

Nearest Neighbor Distance Between Nucleation Sites

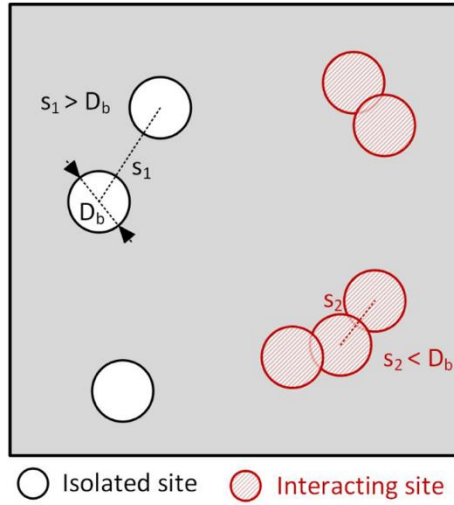


The probability distribution function for distance between nearest neighbors if there are N points randomly distributed on a surface of area A

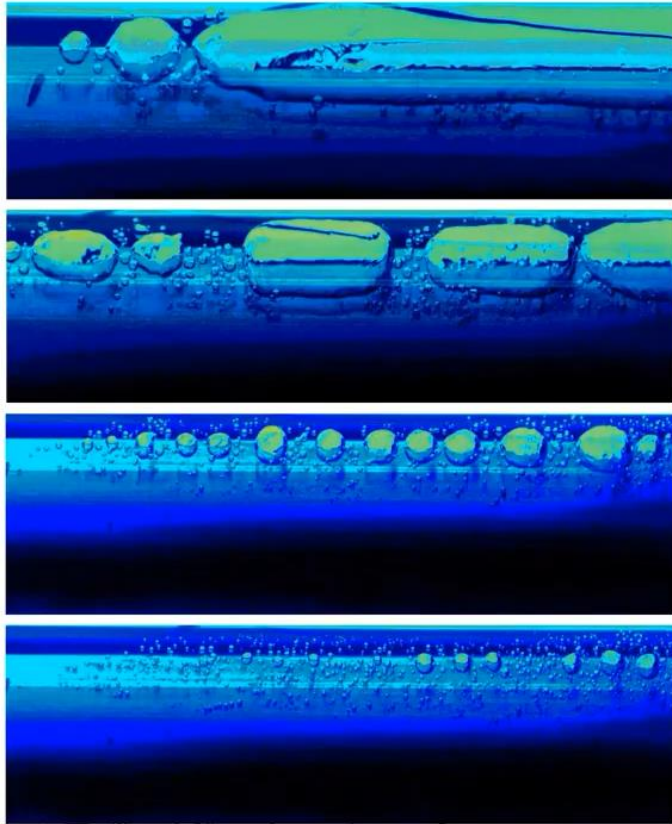
$$f(s) = \frac{2\pi Ns}{A} e^{-\frac{\pi Ns^2}{A}}$$

Rayleigh distribution

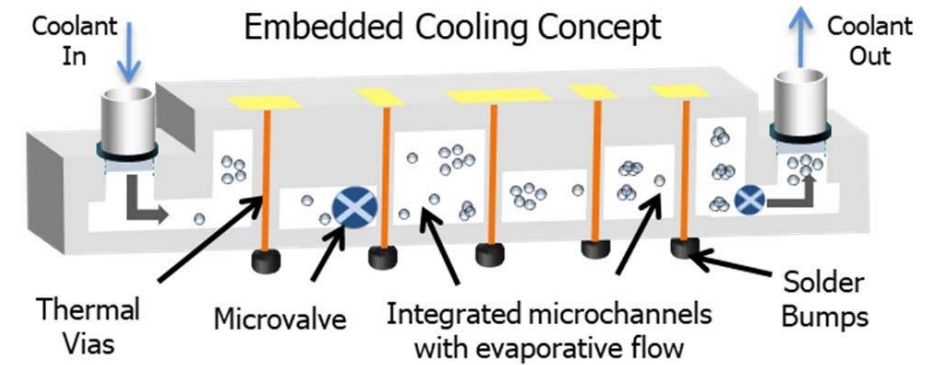
Number of Isolated Bubbles



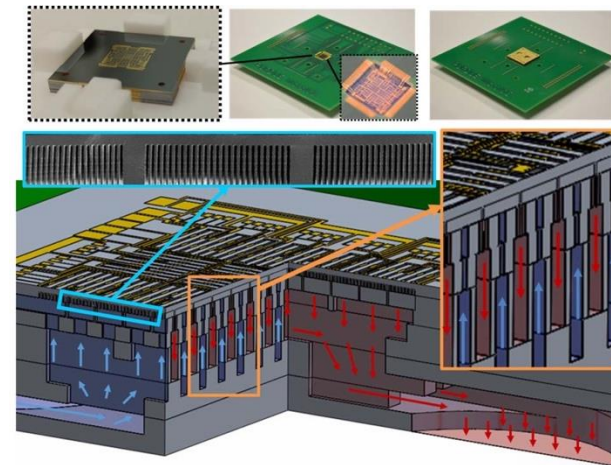
- Force balance model for CHF
- Effect of surface parameters on CHF
- Statistical approach for CHF



Primarily explored for high flux electronics cooling



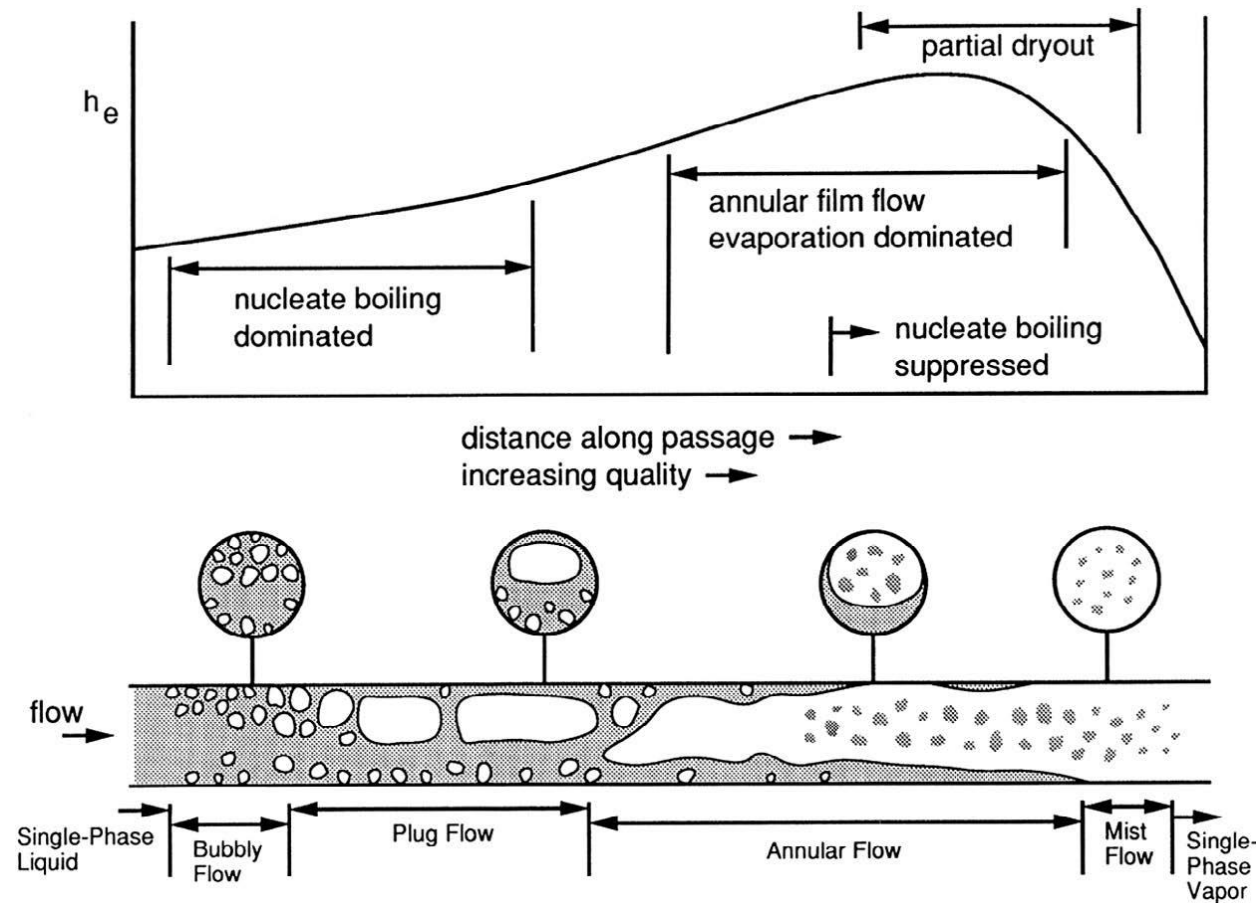
DOI: 10.1109/TCPMT.2021.3111114



Different regimes in flow boiling
(slowed down 50 times)

<https://www.youtube.com/shorts/HNcp7zDtwx8?feature=share>

International Journal of
Heat and Mass Transfer
117 (2018) 319–330



Bubbly flow: discrete bubbles dispersed in continuous liquid phase

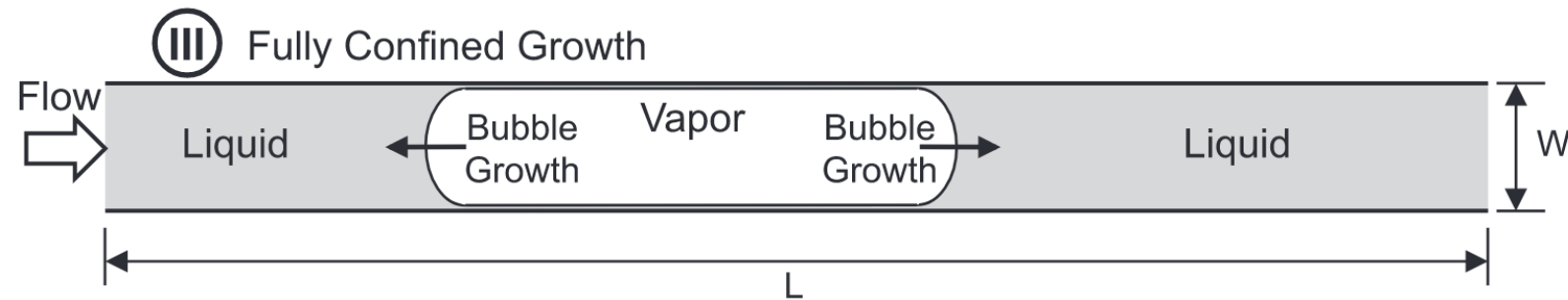
Plug flow: coalescence of small bubbles produces large bubbles flowing in the upper portion

Annular flow: most liquid flowing along the wall and gas flowing in the central core

Mist flow: discrete droplets dispersed in continuous vapor phase

Figure 12.1 in Carey

Oscillations in Microchannel Flow Boiling



Bubble expansion toward the inlet causing density wave oscillation in microchannels

Flow rate reduced (potentially leading to backflow), which gives rise to further vapor expansion toward the inlet

Eventually, liquid rushes back into the channel when sufficient pressure is built up

Vapor Backflow Instability

60 fps; real time duration: 210 ms

Credit: Mark
Schepperle